MATH 1ST 3.3 #s 7, 10, 13, 16, 26, 29, 35, 36

#s 7-44 Follow Steps 1-7, pg 214, to graph

NOTE: STEP 2: "Cross or Touch" is plenty

STEP 5: Testing "above or below" is a good check, but main thing is touch or cross to see you through.

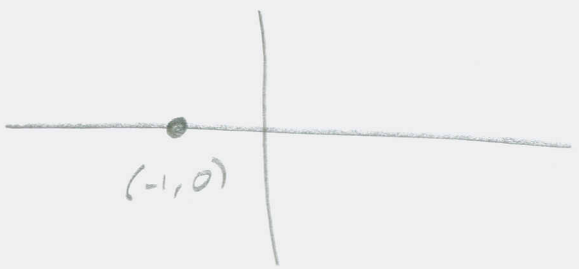
(7) $R(x) = \dfrac{x+1}{x(x+4)}$

(1) $\mathcal{D} = \{x \mid x \neq 0 \text{ & } x \neq -4\}$

y-int: NONE ($x = 0 \notin \mathcal{D}$.)

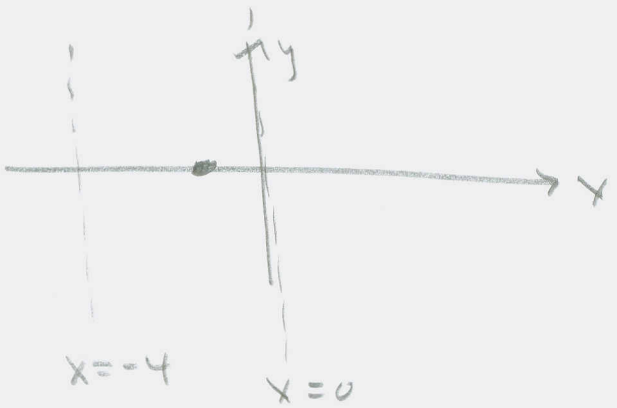
(2) Zeros: $x+1=0$

$\Rightarrow x = -1$



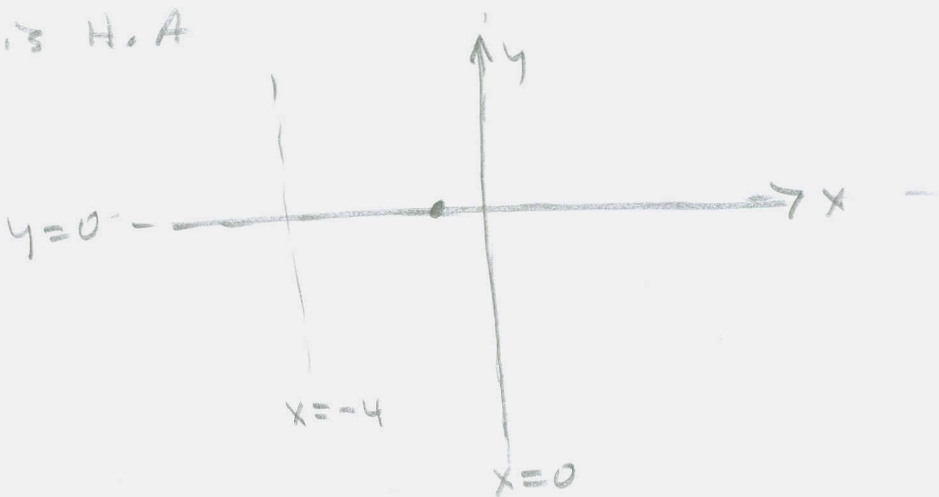
(3) V.A.: $x=0$

$x=-4$



(4) R is PROPER:

$y=0$ is H.A



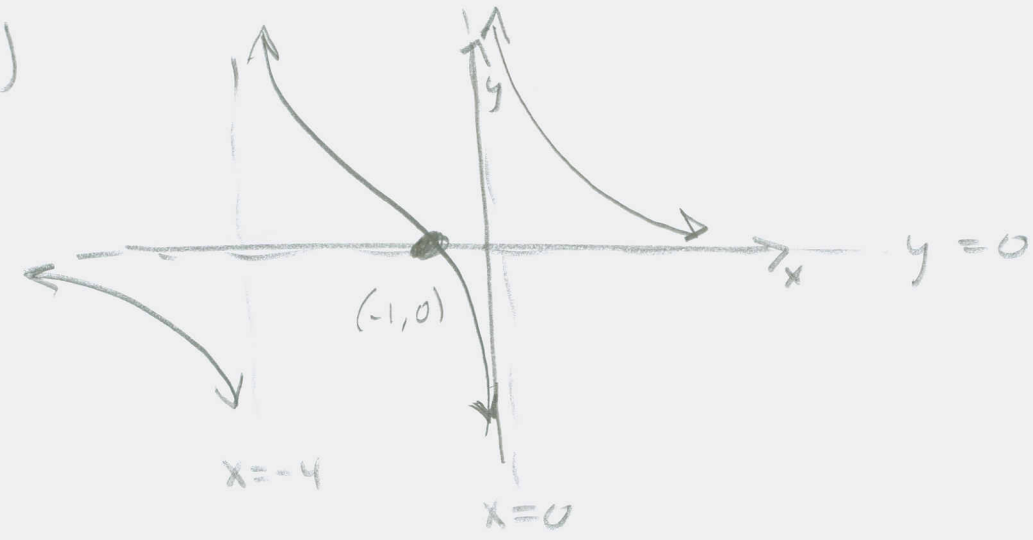
(5)

$x=-4$	$m=1$	cross
$x=-1$	$m=1$	cross
$x=0$	$m=1$	cross

E.B. Test $x=1$: $R(1)=\frac{1+1}{1(1+4)}=\frac{2}{5}$ is + ABOVE

(6) See cross / touch, above

(7)



(10) $R(x)=\frac{2x+4}{x-1}$

$\mathcal{D}=\mathbb{R}\setminus\{1\}=\{x\mid x\neq 1\}$

$x=1$ is V.A.

$R(x)=0 \Rightarrow 2x+4=0$

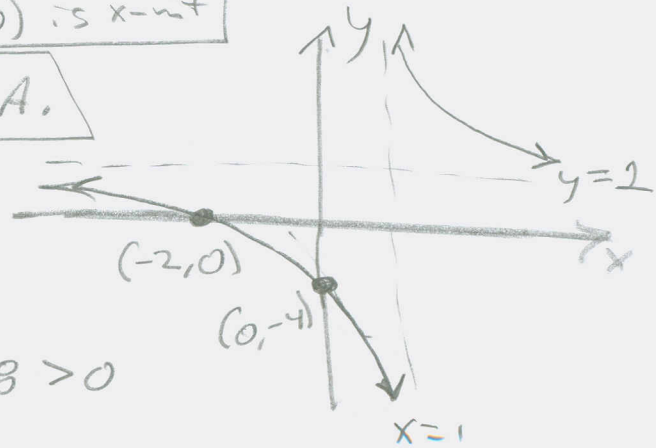
$\Rightarrow x=-2 \rightsquigarrow$ (-2,0) is x-int

H.A.: $y=\frac{2}{1}=2$ $\Rightarrow y=2$ is H.A.

$x=-2$	$m=1$	cross
$x=1$	$m=1$	cross

E.B.: $R(2)=\frac{2(2)+4}{2-1}=\frac{8}{1}=8>0$

$R(0)=\frac{4}{-1}=-4 \rightsquigarrow$ (0,-4) is y-int



(13) $P(x) = \dfrac{x^4+x^2+1}{x^2-1}$

D: Need $x^2-1 \neq 0$

$\Rightarrow x \neq \pm 1 \Rightarrow$

$D = \{x \mid x \neq \pm 1\}$

Also, $x=1$, $x=-1$ are V.A.

Zeros of $P(x)$:

$x^4 + x^2 + 1 = 0$

Let $u = x^2$. Then

$u^2 + u + 1 = 0$

$a=1, b=1, c=1 \Rightarrow$

$b^2 - 4ac = 1^2 - 4(1)(1) = -3$

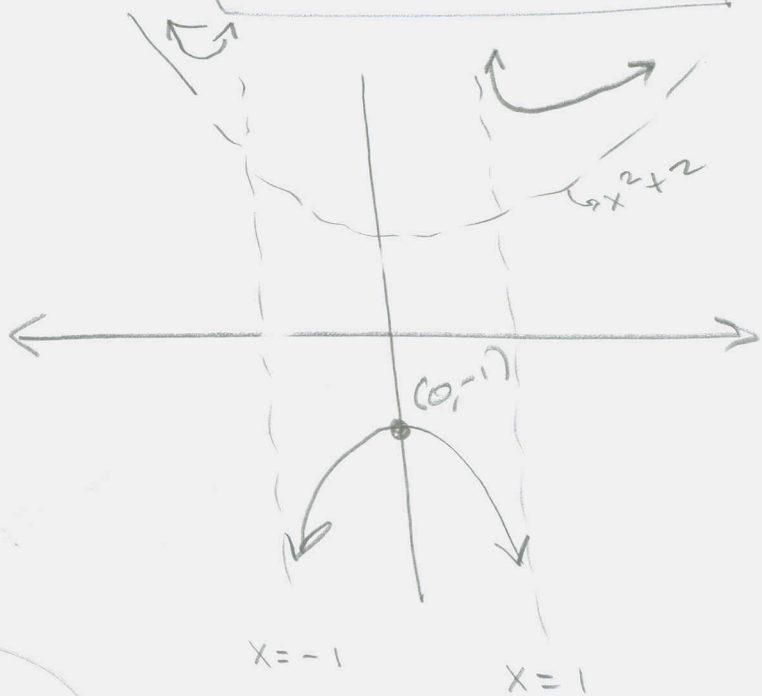
No real sol'n for u

$\Rightarrow$ No soln for x.

$\Rightarrow$ No x-int.

H.A.: None

O.A.: We may not cover these.



$x = -1$ $m = 1$ cross

$x = 1$ $m = 1$ cross

$$P(x) = \frac{x^4+x^2+1}{(x-1)^1(x+1)^1}$$

$$P(2) = \frac{2^4+2^2+1}{(2-1)(2+1)} = \frac{16+4+1}{(1)(3)} > 0$$

$$x^2-1 \overline{)\, x^4+0x^3+x^2+0x+1}$$

x^2+2 r 3

$-(x^4 \quad -x^2)$

$2x^2+0x+1$

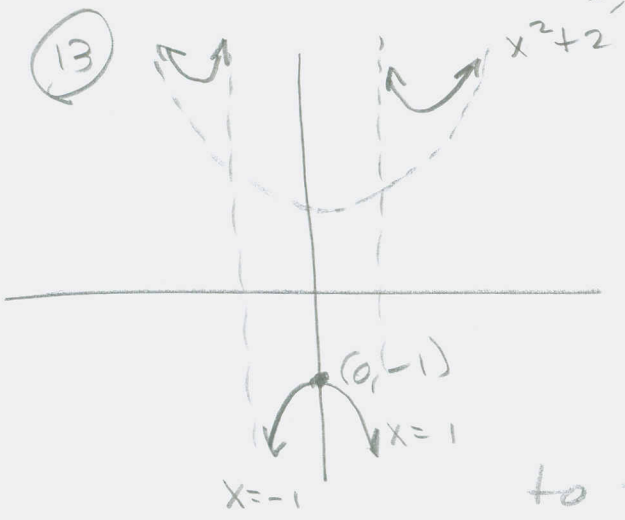
$-(2x^2 \quad -2)$

$r = 3$

So O.A. is x^2+2

Re-Sketch on next page

(13)



To do E.B.:

$P(2) = \frac{21}{3} = 7 \rightsquigarrow (2,7)$

This gets you started as to where $P(x)$ is off to the right of then "touch/cross"

(16) $G(x) = \frac{x^3+1}{x^2+2x} = \frac{(x+1)(x^2-x+1)}{x(x+2)}$

$\mathcal{D} = \mathbb{R} \setminus \{-2, 0\}$

V.A.: $x=-2$, $x=0$ (Nothing cancels)

x-int: $x+1=0 \Rightarrow x=-1 \rightsquigarrow (-1,0)$ = x-int.

$x=-2$	$m=1$	Cross
$x=-1$	$m=1$	
$x=0$	$m=1$	

E.B.: Has an oblique Asymptote

$$\begin{array}{r} x-2 \text{ r } 4x+1 \\ x^2+2x \overline{)\, x^3+0x^2+0x+1} \\ -(x^3+2x^2) \\ \hline -2x^2+0x+1 \\ -(-2x^2-4x) \\ \hline r = 4x+1 \end{array}$$

O.A.: $y = x-2$

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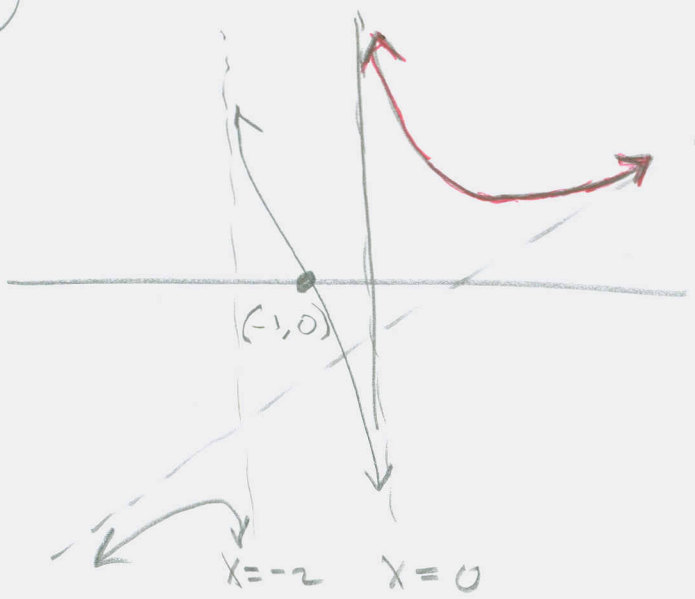
(16)



$$G(1) = \frac{1^3+1}{1^2+2(1)}$$

$$= \frac{2}{1+2} = \frac{2}{3} > 0$$

This gives us the general location of the right $\frac{1}{3}$ of the graph (in red)

(26) $F(x) = \dfrac{x^2+3x+2}{x-1}$

upstairs higher degree than downstairs. Another O.A.

Numerator denominator

$$\begin{array}{r|rrr} 1 & 1 & 3 & 2 \\ & & 1 & 4 \\ \hline & 1 & 4 & 6 \end{array}$$

This says $\dfrac{x^2+3x+2}{x-1} = (x+4) + \dfrac{6}{x-1}$

→ oblique Asymptote

O.A. = $y = x+4$

$D: \{x \mid x \neq 1\}$

$x = 1$ is V.A.

$$F(x) = \frac{(x+2)(x+1)}{x-1}$$

zeros: $x^2 + 3x + 2 = 0$

$\Rightarrow (x+2)(x+1) = 0$

$\Rightarrow x = -1$ or $x = -2$ are the zeros.

x-int: $(-2, 0)$, $(-1, 0)$

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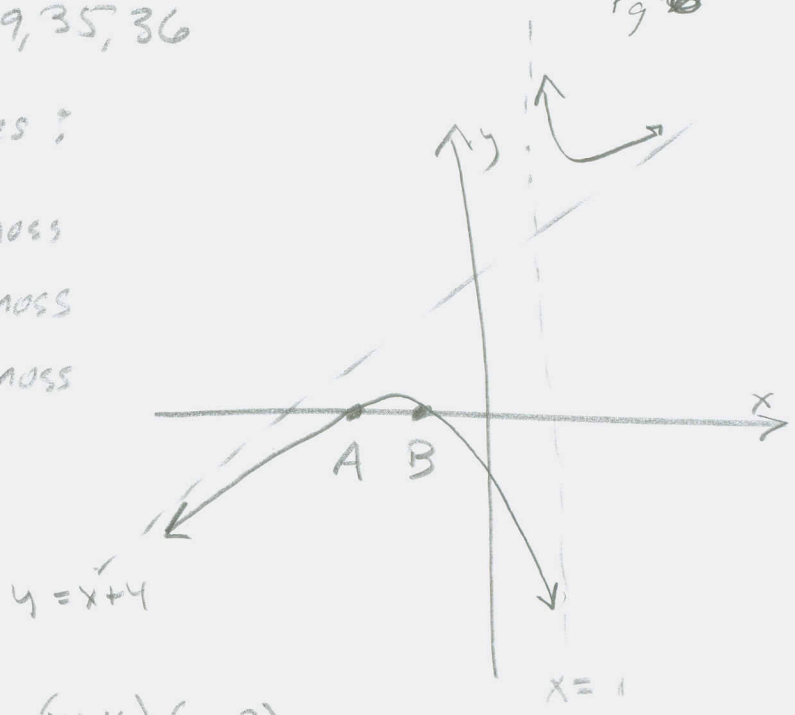
(26) Key or Critical Values:

x-int	$x = -2$	$m = 1$	Cross
	$x = -1$	$m = 1$	Cross
V.A.	$x = 1$	$m = 1$	Cross

$A = (-2, 0)$

$B = (-1, 0)$



(29) $F(x) = \dfrac{x^2+x-12}{x+2} = \dfrac{(x+4)(x-3)}{x+2}$

deg(num) > deg(denom) $\Longrightarrow$ O.A. & No H.A.

$D: \{x \mid x \neq -2\}$ OR $\mathbb{R} \setminus \{-2\}$

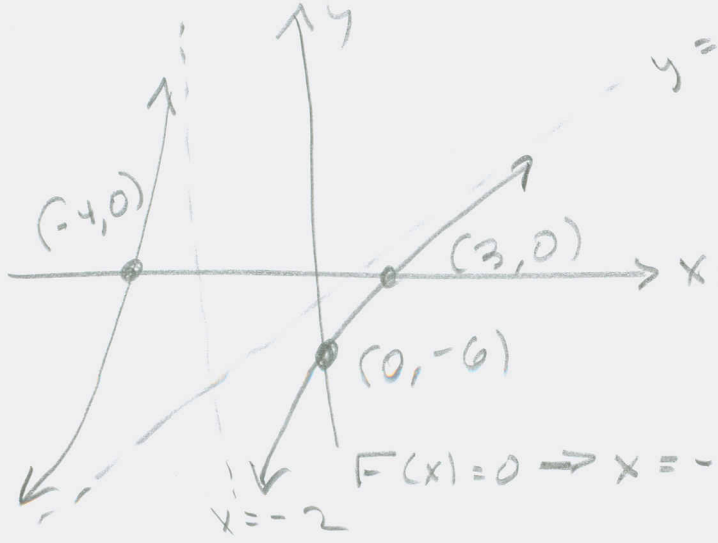
OR $(-\infty, -2) \cup (-2, \infty)$ & $x = -2$ is V.A.

V.A.: $x = -2$

O.A.:

$$\begin{array}{r|rrr} -2 & 1 & 1 & -12 \\ & & -2 & 2 \\ \hline & 1 & -1 & -10 \end{array}$$

$y = x - 1$ is O.A.



$F(x) = 0 \rightarrow x = -4, x = 3$

$x = -4$	$m = 1$	Cross
$x = -2$	$m = 1$	Cross
$x = 3$	$m = 1$	Cross

$F(4) = \dfrac{(8)(1)}{6} > 0$

$F(0) = \dfrac{-12}{2} = -6$

$(0, -6)$ is y-int

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(35) $R(x) = \dfrac{6x^2-7x-3}{2x^2-7x+6} = \dfrac{6x^2-9x+2x-3}{2x^2-4x-3x+6}$

$= \dfrac{3x(2x-3)+1(2x-3)}{2x(x-2)-3(x-2)} = \dfrac{(2x-3)(3x+1)}{(x-2)(2x-3)}$

Ahh! The $2x-3$'s cancel! This will leave a hole @ $x=\frac{3}{2}$, but otherwise not have anything to do with the graph. Let's get the hole outta the way & then work with $\dfrac{3x+1}{x-2}$.

Hole at $x=\frac{3}{2} \Rightarrow y = \dfrac{3(\frac{3}{2})+1}{\frac{3}{2}-2} = \dfrac{\frac{9}{2}+1}{-\frac{1}{2}} = \dfrac{\frac{11}{2}}{-\frac{1}{2}}$

$= \left(\frac{11}{2}\right)\left(-\frac{2}{1}\right) = -11 \Rightarrow$ **Hole is @ $\left(\frac{3}{2}, -11\right)$**

Now, $\dfrac{3x+1}{x-1} = R(x) \quad (x \neq \frac{3}{2})$

O.A.: None (Thank goodness!)

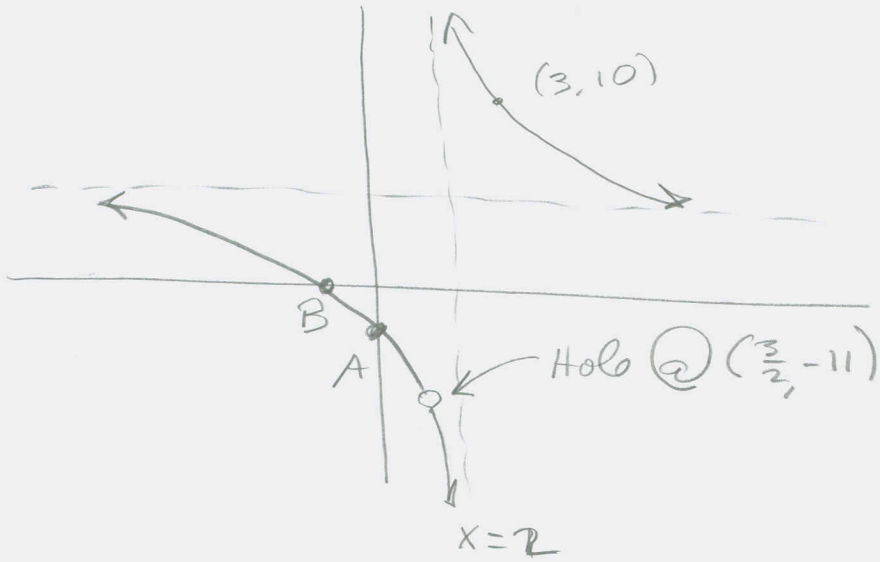
H.A.: $y = \frac{3}{1} =$ **$3 = y$ is H.A.**

$\mathcal{D}$: Need $x \neq \frac{3}{2}$ and Need $x-1 \neq 0 \Rightarrow x \neq 1$

$\rightarrow \mathcal{D} = \{x \mid x \neq 1 \text{ and } x \neq \frac{3}{2}\}$

V.A.: $x = 1$ ($x = \frac{3}{2}$ is where the hole is.)

(35) Put it together



$R(3) = \frac{3(3)+1}{3-2} = \frac{10}{1} = 10$ ✓

x-int: $3x+1=0 \Rightarrow x = -\frac{1}{3} \rightsquigarrow \left(-\frac{1}{3}, 0\right) = A$

y-int: $R(0) = \frac{3(0)+1}{0-2} = -\frac{1}{2} \rightsquigarrow \left(0, -\frac{1}{2}\right) = B$

(36) $R(x) = \frac{8x^2+26x+15}{2x^2-x-15} = \frac{8x^2+20x+6x+15}{2x^2-6x+5x-15}$

$= \frac{4x(2x+5)+3(2x+5)}{2x(x-3)+5(x-3)} = \frac{(2x+5)(4x+3)}{(x-3)(2x+5)}$ Hole @ $x = -5/2$

$= \frac{4x+3}{x-3}$ $\left(x \neq -\frac{5}{2}\right)$ is in lowest terms.

H.A.: $y = 4$

V.A.: $x = 3$

Hole: $R\left(-\frac{5}{2}\right) = \frac{4\left(-\frac{5}{2}\right)+3}{-\frac{5}{2}-3}$

$= \frac{-10+3}{\frac{-5-6}{2}} = \frac{-7}{\frac{-11}{2}} = \frac{14}{11}$

$\rightsquigarrow$ Hole @ $\left(-5/2, 14/11\right)$

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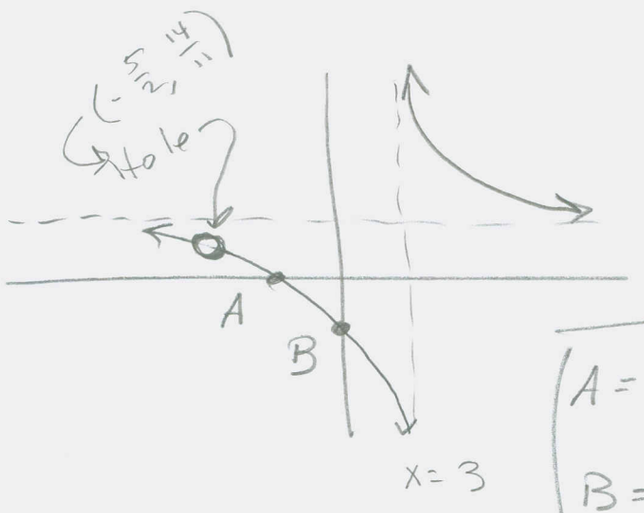
(36)



$A = (-\frac{1}{3}, 0)$

$B = (0, -\frac{1}{2})$